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Depolarization in spectroscopic ellipsometry

Bachelor thesis

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Zadání bakalářské práce

Student: **Jan Chochol**

Studijní program: B3942 Nanotechnologie

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Téma: Depolarizace ve spektroskopické elipsometrii
Depolarization in spectroscopic ellipsometry

Zásady pro vypracování:

Cílem bakalářské práce je měření depolarizace vzorků metodami fázově modulační spektroskopické elipsometrie. Práce bude vypracována ve třech etapách:
1. Popis depolarizačních vlastností vzorku - zahrnuje rešerši literatury, popis aparátem Muellerovy matice, definice depolarizačního faktoru.
2. Návrh metody měření pomocí fázově modulační elipsometrie - maticový popis elipsometru, návrh zónového středování, odstranění neideálnosti elipsometru pro izotropní vzorky.
3. Experimentální ověření navržené metody na depolarizujících a nedepolarizujících vzorcích a její modelování.

Aim of this work is to measure the sample depolarizations using a phase-modulation spectroscopic ellipsometry. It will be solved in three steps:
1. Description of sample depolarization properties, which is based on a literature search and includes description using Mueller matrices, and definition of depolarization factor.
2. Proposition of a measurement method using phase-modulation ellipsometry - matrix description of ellipsometer, zone averaging algorithm, and elimination of ellipsometer imperfections for isotropic samples.
3. Experimental verification, demonstration, and modelling of proposed method using depolarizing and nondepolarizing samples.

Seznam doporučené odborné literatury:

1. R. M. Azzam, N. M. Bashara, Ellipsometry and polarized light, North-Holland, Amsterdam 1977.
2. Ch. Brosseau, Fundamentals of polarized light: A statistical optics approach, John Willey & Sons, New York 1998.
3. články v odborných mezinárodních časopisech

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Vedoucí bakalářské práce: **doc. Dr.Mgr. Kamil Postava**

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vedoucí katedry

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prorektor pro studium

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Declaration

I declare I elaborated this thesis by myself. All literary sources and publications I have used had been cited.

Ostrava, May 21, 2010 ……………………………………………

I would like to thank doc. Dr. Mgr. Kamil Postava for his guidance, help, will and motivation by writing this thesis.

Abstrakt

Spektroskopická elipsometrie je metoda široce používaná v průmyslu i výzkumu k optické charakterizaci různých systémů, ať už vrstevnatých struktur, metamateriálů, biologických vzorků nebo roztoků. Depolarizační jevy mohou tato měření narušovat a proto je vhodné vědět, jak je detekovat a popř. odstranit. V této práci je pomocí aparátu Muellerových matic popsán spektroskopický elipsometr a pomocí fázové modulace je ukázáno, jak lze depolarizaci detekovat. Pro minimalizaci chyb je využito zónového středování.

Klíčová slova: elipsometrie, spektroskopická elipsometrie, depolarizace, Muellerovy matice, stupeň polarizace

Abstract

Spectroscopic ellipsometry is method widely used in research and industry for optical characterization of various systems, such as layered materials, metamaterials, biological samples or solutions. Depolarization effects can cause problems with data measurement a therefore it is necessary to know how to detect them and preferably how to remove them. In this work is the spectroscopic ellipsometer described by Mueller matrix calculus and it is shown, how the depolarization can be detected through the phase modulation. Error minimalization is done by zone averaging.

Key words: ellipsometry, spectroscopic ellipsometry, depolarization, Mueller matrix, degree of polarization.

Contents

List of shortcuts, abbreviations and notation

Vectors, matrices ... bold and italic

Scalars ... italic

$\boldsymbol{J}^{+}$ = Hermitian adjoint

$\boldsymbol{J}^{\mathrm{T}}$ = Transposition

a^{*} = Conjugate

< > = Spacial or temporal averaging

r_s, r_p, t_s, t_p = Fresnel coefficients

Ψ, Λ = Ellipsometric angles

PEM = Photoelastic modulator

DOP = Degree of polarization

PI = Polarization index

1. Introduction

Thin films system are technological wonder and are widely used throughout the science and technology world and have great importance in many fields of research interest, such as nanotechnology, microelectronics, photonics, optoelectronics, energy, etc[1].Therefore it is necessary to have methods for characterization of such systems. Ellipsometry is one of the best of them. It is extremely powerful method that allows fast and accurate characterization of not just thin films systems, but also biological samples or solutions[2]. Ellipsometry has many advantages, it provides reproducible, non-destructive measurement with accuracy in few nanometers with quite simple experimental arrangement. It can be also used in situ, ex situ, in high pressure or magnetic environment[3]. The principle of ellipsometry is to reflect the polarized light from the measured sample and detect the intensity dependent of several factors. The light reflected from boundary has generally elliptical polarization, hence the method is called ellipsometry. The data obtained from ellipsometry are not usually not interesting, they must be compared with model of the measured sample and the parameters, such as film thickness, dielectric function, surface roughness are determined by so called fitting, which is well understood exercise in science and engineering.

However, some of the samples might exhibit effects, which might disturb the correct measurement. One of them is effect called depolarization. It occurs when the reflected light is not totally polarized. This may happen for example when we have thin film system on thick substrate[4]. So it would be advantageous to know the causes of depolarization and how to deal with them. Currently, there are several approaches on how to deal with depolarization, ranging from special experimental setup[5] to elaborated mathematical algorithms[6].

This thesis proposes a simple method for detection of depolarization by phase modulation. In the first section, the mathematical background of light polarization is outlined and the degree of polarization a polarization index is defined. Second section describes the spectroscopic ellipsometer using Mueller formalism and the calculation of detected parameters. This section is also postulated how to minimize the measurement errors. The next section verifies the assumptions introduced in previous sections by measurement on slightly depolarizing sample.

2. Theoretical background

In this section we summarize basic knowledge about light and its properties needed for our experimental setup and understanding of ongoing effects.

2.1. Mathematical description of light polarization

Light is physical phenomenon, generally described by four approaches: ray, wave, electromagnetic and quantum [7]. We shall use the electromagnetic approach, because the ray and the wave model are not able to describe polarization effects in our experiment and implications given by the quantum optics are not observable.

Electromagnetic model postulates, that light can be described as combination of four vectors fields: the electric field $\boldsymbol{E}$, the magnetic field $\boldsymbol{H}$, the electric flux density $\boldsymbol{D}$ and the magnetic flux density $\boldsymbol{B}$. These vector fields are related to each other by the Maxwell's equations[8].

$$\nabla\times\boldsymbol{H}=\frac{\partial\boldsymbol{D}}{\partial t}+j,\quad \nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t},\quad \nabla\cdot\boldsymbol{D}=\rho,\quad \nabla\cdot\boldsymbol{B}=0, \tag{2.1}$$

where t denotes time, j is the current density, and ρ charge density.

To describe the interaction of light with matter, we need to complete equations (2.1) with the material equations

$$\boldsymbol{D}=\varepsilon\varepsilon_0\boldsymbol{E},\quad \boldsymbol{B}=\mu\mu_0\boldsymbol{H},\; j=\sigma\boldsymbol{E}, \tag{2.2}$$

where ε is the relative permittivity of the medium, ε_0 is the permittivity of vacuum ($8.85\ 10^{-12}$ $\mathrm{Fm^{-1}}$), μ is the permeability of medium, μ_0 is the permeability of vacuum ($4\pi 10^{-7}\,\mathrm{Hm^{-1}}$) and σ denotes the conductivity of medium.

Furthermore, after application material equations (2.2) and curl on Eq. (2.1) we obtain the wave equation

$$\nabla^2\boldsymbol{E}-\frac{1}{\varepsilon\mu}\frac{\partial^2\boldsymbol{E}}{\partial t^2}=0. \tag{2.3}$$

Moreover, we consider the solution of wave equation to be monochromatic wave with the frequency ω.

$$\boldsymbol{E}(z,t)=\Re\left\{A(r)\exp\left[i\omega(t-\frac{z}{c})\right]\right\}, \tag{2.4}$$

where $\Re$ denotes the real part and A is the complex envelope.

$$A=A_x\hat{\boldsymbol{x}}+A_y\hat{\boldsymbol{y}} \tag{2.5}$$

Eq. (2.5) shows, that complex envelope is a vector composed out of two complex components

$$A_x=a_x e^{i\varphi x}, \quad A_y=a_y e^{i\varphi y}. \tag{2.6}$$

Now we can combine equations (2.4-2.6) and express the following formulas:

$$\begin{aligned} E_x&=a_x\cdot\cos\left[2\pi f\left(t-\frac{z}{c}\right)+\varphi_x\right],\\ E_y&=a_y\cdot\cos\left[2\pi f\left(t-\frac{z}{c}\right)+\varphi_y\right]. \end{aligned} \tag{2.7}$$

Eq. (2.7) represents parametric expression of **elliptic polarization**

$$\frac{E_x^{\ 2}}{a_x^2}+\frac{E_y^{\ 2}}{a_y^2}-2\cos\varphi\frac{E_x E_y}{a_x a_y}=\sin^2\varphi, \tag{2.8}$$

where $\varphi = \varphi_y - \varphi_x$ is the phase difference.

The tip of electric-field vector rotates periodically and thus creating the ellipse. The size of this ellipse is determined by the intensity of the wave, which is proportional to $|A_x|^2+|A_y|^2=a_x^2+a_y^2$. There are some special cases which must be discussed. First, if one of the components vanishes, or the phase difference is equal to π or 0, Eq. (3.8) changes to equation of straight line, therefore creating **linearly polarized light**. The other interesting case originate from phase difference φ = ±π/2 and equal amplitudes a_x=a_y. The ellipse changes to a circle and creates **circularly polarized light**.

2.2. Matrix representation

There are several approaches for definition of polarized light and polarization optics. The first (although not first historically) and the simplest approach used is the Jones formalism. It uses two-element vector for light and 2x2 matrix for optical components. Although quite simple and easy to calculate, it provides extremely powerful tool for dealing with polarization and all the other approaches mentioned can relate to it. However, it is not sufficient for our cause, because it can only deal with amplitudes and therefore can describe only totally polarized light (as will be shown later). Historically first description of polarized light was by the Stokes four-element vector. This formalism suits our needs, because it operates with intensities. And for description of optical components we shall use the Mueller 4x4 matrix. 4x4 Coherency-transforming matrix and coherency vector are also optional, but Mueller matrix is more schematic and easier to deal with.

2.2.a) Jones formalism

As shown in Eq. (2.6), electromagnetic wave traveling along z-axis can be described by 2 complex envelopes, A_x and A_y. These two parameters compose the Jones vector.

$$\boldsymbol{J}=\begin{bmatrix} A_x \\ A_y \end{bmatrix} \tag{2.9}$$

we can represent any totally polarized light by this vector, as shown in Table (2.I)

Name	**Jones vector**
Linearly polarized in axis x	$\boldsymbol{J}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
Linearly polarized in axis y	$\boldsymbol{J}=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Linearly polarized at +45º	$\boldsymbol{J}=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
Linearly polarized at -45º	$\boldsymbol{J}=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
Linearly polarized at α	$\boldsymbol{J}=\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$
Right-handed circular	$\boldsymbol{J}=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$
Left-handed circular	$\boldsymbol{J}=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$

Table (2.I) – Jones vectors for several polarization states

In order to calculate intensity, which is the measurable quantity, we need to multiply the Jones vector $\boldsymbol{J}$ by its Hermitian adjoint $\boldsymbol{J}^+$.

$$\boldsymbol{J}^+=\left[A_x^*, A_y^*\right] \tag{2.10}$$

$$I=\boldsymbol{J}^{+}\dot{\boldsymbol{J}}=A_x A_x^{*}+A_y A_y^{*}=a_x^2(e^{i\varphi_x}.e^{-i\varphi_x})+a_y^2(e^{i\varphi_y}.e^{-i\varphi_y})=a_x^2+a_y^2 \qquad (2.11)$$

For description of polarization devices we use the 2x2 Jones matrix formalism. It is based upon this model: plane wave of arbitrary polarization transmits through optical system that alters the wave's polarization. The input and output wave have two components: A_{1x}, A_{1y} and A_{2x}, A_{2y}, respectively. Polarization device changes the input wave polarization into output using four complex factors T_{ij}, i,j=1,2.

$$\begin{aligned} A_{2\mathrm{x}}&=T_{11}A_{1\mathrm{x}}+T_{12}A_{1\mathrm{y}} \\ A_{2\mathrm{y}}&=T_{21}A_{1\mathrm{x}}+T_{22}A_{1\mathrm{y}} \end{aligned} \qquad (2.12)$$

Eq. (2.12) can be easily written in matrix form.

$$\overbrace{\begin{bmatrix} A_{2\mathrm{x}} \\ A_{2\mathrm{y}} \end{bmatrix}}^{\boldsymbol{J_2}}=\overbrace{\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}}^{\boldsymbol{T}}.\overbrace{\begin{bmatrix} A_{1\mathrm{x}} \\ A_{1\mathrm{y}} \end{bmatrix}}^{\boldsymbol{J_1}}, \qquad (2.13)$$

where $\boldsymbol{J_1}$ is the Jones vector of incident wave, $\boldsymbol{J_2}$ Jones vector of output wave and $\boldsymbol{T}$ is the Jones matrix of polarization device.

2.2.b) Coherency vector and coherency-transforming matrix

In order to calculate the coherency vector, we must first determine the coherency matrix by creating Kronecker product of the Jones vector and its Hermitian adjoint.

$$\boldsymbol{J_c}=\begin{bmatrix} \langle E_x E_x^{*}\rangle & \langle E_x E_y^{*}\rangle \\ \langle E_y E_x^{*}\rangle & \langle E_y E_y^{*}\rangle \end{bmatrix}=\begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix}, \qquad (2.14)$$

where <> denotes spatial or temporal averaging.

Then, the coherency vector consists of the elements of the coherency matrix.

$$\boldsymbol{J_{co}}=\begin{pmatrix} J_{xx} \\ J_{xy} \\ J_{yx} \\ J_{yy} \end{pmatrix} \qquad (2.15)$$

Coherency-transform matrix describing the non-depolarizing device is also defined by the Kronecker product, this time for Jones matrix and its Hermitian adjoint.

$$\boldsymbol{T_c}=\begin{bmatrix} T_{11}T_{11}^* & T_{11}T_{12}^* & T_{12}T_{11}^* & T_{12}T_{12}^* \\ T_{11}T_{21}^* & T_{11}T_{22}^* & T_{12}T_{21}^* & T_{12}T_{22}^* \\ T_{21}T_{11}^* & T_{21}T_{12}^* & T_{22}T_{11}^* & T_{22}T_{12}^* \\ T_{21}T_{21}^* & T_{21}T_{22}^* & T_{22}T_{21}^* & T_{22}T_{22}^* \end{bmatrix} \tag{2.16}$$

The output coherency vector then satisfies

$$\boldsymbol{J_{oco}}=\boldsymbol{T_c}\cdot\boldsymbol{J_{ico}} \tag{2.17}$$

2.2.c) Stokes vector

Stokes vector is set of four real numbers (S_0, S_1, S_2, S_3), which are directly linked to the intensity of light. Each of these components has its own distinctive meaning: S_0 denotes the total intensity, S_1 is the difference between x-polarization and y-polarization, S_2 is the difference between polarization at 45º and polarization at -45º and S_3 is the difference between right-handed circular polarization and left-handed circular polarization. These parameters can be calculated from Jones vector, but the Stokes vector will describe only totally polarized light.

$$\boldsymbol{S}=\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}=\begin{pmatrix} I_0 \\ I_x-I_y \\ I_{45}-I_{-45} \\ I_{RCP}-I_{LCP} \end{pmatrix}=\begin{pmatrix} A_x A_x^*+A_y A_y^* \\ A_x A_x^*-A_y A_y^* \\ 2\Re\{A_x^* A_y\} \\ 2\Im\{A_x^* A_y\} \end{pmatrix} \tag{2.18}$$

For totally polarized light applies that $S_0^2=S_1^2+S_2^2+S_3^2$.

Name	Stokes vector
Linearly polarized in axis x	$\boldsymbol{S}=[1,1,0,0]^T$
Linearly polarized in axis y	$\boldsymbol{S}=[1,-1,0,0]^T$
Linearly polarized at +45º	$\boldsymbol{S}=[1,0,1,0]^T$
Linearly polarized at -45º	$\boldsymbol{S}=[1,0,-1,0]^T$
Linearly polarized at α	$\boldsymbol{S}=[1,\cos^2\alpha-\sin^2\alpha,\sin 2\alpha,0]^T$
Right-handed circular	$\boldsymbol{S}=[1,0,0,1]^T$
Left-handed circular	$\boldsymbol{S}=[1,0,0,-1]^T$
Unpolarized light	$\boldsymbol{S}=[1,0,0,0]^T$

Table (2.II) – Stokes vectors for several polarization states

As shown in Table (2.II), Stokes vector can be used for description of unpolarized and partially polarized light. This will be discussed later on.

2.2.d) Mueller matrix

Mueller matrix is used, when we want to describe light by Stokes vector. Incident light S_i interacts with optical device and its transformed to output vector S_o

$$\begin{pmatrix} S_{o0} \\ S_{o1} \\ S_{o2} \\ S_{o3} \end{pmatrix} = \begin{pmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{pmatrix} \cdot \begin{pmatrix} S_{i0} \\ S_{i1} \\ S_{i2} \\ S_{i3} \end{pmatrix}. \tag{2.19}$$

It is possible for non-depolarizing systems to determine the Mueller matrix from the Jones matrix through the following process: We calculate the coherency-transform matrix and than apply the matrix $\boldsymbol{A}$

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -i & i & 1 \end{pmatrix}, \tag{2.20}$$

so the Mueller matrix will be

$$\boldsymbol{M} = \boldsymbol{A}\boldsymbol{T}_c\boldsymbol{A}^{-1}. \tag{2.21}$$

2.2.e) Optical components

The following table shows commonly used polarization devices in their matrix description and verbal explanation of their function.

Name of polarization device	Jones matrix representation / Coherency-transform matrix representation / Mueller matrix representation / Description
Linear	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

polarizer (x-axis)	$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$	
	$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$	
	Linear polarizer changes any incident light into linearly polarized, in other words, it eliminates its orthogonal polarization. Two linear polarizers are often used in various optical systems in combination called polarizer-analyzer.	
Linear polarizer (y-axis)	$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$	
	$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$	
	$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$	
	Linear polarizer for y-axis is here for demonstration of different representation in every formalism.	
Wave retarder	$$\begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix}$$	
	$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\Gamma} & 0 & 0 \\ 0 & 0 & e^{-i\Gamma} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$	

	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\Gamma & -\sin\Gamma \\ 0 & 0 & \sin\Gamma & \cos\Gamma \end{bmatrix}$
	Wave retarder changes phase difference between the two complex envelopes by the angle Γ.
Photoelastic modulator (PEM)	$\begin{bmatrix} e^{i\frac{\varphi}{2}} & 0 \\ 0 & e^{-i\frac{\varphi}{2}} \end{bmatrix}$
	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\varphi} & 0 & 0 \\ 0 & 0 & e^{-i\varphi} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\varphi & -\sin\varphi \\ 0 & 0 & \sin\varphi & \cos\varphi \end{bmatrix}$
	PEM is wave retarder with changing retardation angle, caused by birefringence and photoelastic effect. The retardation angle oscillates at frequency f=2π/ω and its described by following equation: $\varphi = \varphi_0 + \sin(\omega t)$, where φ_0 is the residual birefringence. Its function in optical systems is to create time-dependent polarization states – linear, circular and elliptical.
Polarization rotator	$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
	$\begin{bmatrix} \cos^2\theta & -\cos\theta\sin\theta & -\cos\theta\sin\theta & 1-\cos^2\theta \\ \cos\theta\sin\theta & -\cos^2\theta & -1+\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & -1+\cos^2\theta & \cos^2\theta & -\cos\theta\sin\theta \\ 1-\cos^2\theta & \cos\theta\sin\theta & \cos\theta\sin\theta & \cos^2\theta \end{bmatrix}$
	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2\cos^2\theta-1 & -2\cos\theta\sin\theta & 0 \\ 0 & 2\cos\theta\sin\theta & 2\cos^2\theta-1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
	Polarization rotator changes the linearly polarized wave $\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix}$ into

	$\begin{bmatrix} \cos(\theta_1+\theta) \\ \sin(\theta_1+\theta) \end{bmatrix}$. As the name suggests, it rotates the plane of polarization by the angle θ.
Coordinate transformation	$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
	$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & \cos\theta\sin\theta & 1-\cos^2\theta \\ -\cos\theta\sin\theta & \cos^2\theta & -1+\cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -1+\cos^2\theta & \cos^2\theta & \cos\theta\sin\theta \\ 1-\cos^2\theta & -\cos\theta\sin\theta & -\cos\theta\sin\theta & \cos^2\theta \end{bmatrix}$
	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2\cos^2\theta-1 & 2\cos\theta\sin\theta & 0 \\ 0 & -2\cos\theta\sin\theta & 2\cos^2\theta-1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
	Coordinate transformation is used, when we rotate polarization device by angle θ. In other words, we transform light from one coordinate system into another by simple matrix operations. First, we rotate incident light by angle θ, then let it through our optical component (T) and then return it to our original coordinate system by rotation by $-\theta$, in matrix representation: T`=R($-\theta$).T.R(θ).

Table (2.III) – Commonly used optical components and their matrix representation

Complete optical system can be also represented as one matrix. For example, we have two components, $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ and incident wave $\boldsymbol{A_0}$. Outgoing wave $\boldsymbol{A_2}$ is determined like this:

$$\begin{aligned} \boldsymbol{A}_1 &= \boldsymbol{T}_1 \boldsymbol{A}_{0,} \\ \boldsymbol{A}_2 &= \boldsymbol{T}_2 \boldsymbol{A}_{1,} \\ \boldsymbol{A}_2 &= \boldsymbol{T}_2 \boldsymbol{T}_1 \boldsymbol{A}_{0.} \end{aligned} \tag{2.22}$$

It is easy to see, that combination of n components is product of their multiplication, from last to first

$$\boldsymbol{T} = \boldsymbol{T}_n \boldsymbol{T}_{n-1} \ldots \boldsymbol{T}_2 \boldsymbol{T}_1 . \tag{2.23}$$

2.2.f) Reflection and refraction

This section describes behavior of polarized light at planar boundary of two dielectric materials. All effects in this chapter will be characterized for simplicity by Jones vector and all optical components by Jones matrix.

First, we consider three waves, the incident wave $\boldsymbol{J_1}$, the refracted wave $\boldsymbol{J_2}$ and the reflected wave $\boldsymbol{J_3}$.

$$\boldsymbol{J_1}=\begin{bmatrix} A_{1x} \\ A_{1y} \end{bmatrix}, \quad \boldsymbol{J_2}=\begin{bmatrix} A_{2x} \\ A_{2y} \end{bmatrix}, \quad \boldsymbol{J_3}=\begin{bmatrix} A_{3x} \\ A_{3y} \end{bmatrix} \tag{2.24}$$

Angles between them are determined by Snell's law

$$\begin{gathered} \theta_1=\theta_3 \\ n_1 \sin\theta_1 = n_2 \sin\theta_2 , \end{gathered} \tag{2.25}$$

where θ_1 = angle between incident wave and optical axis, θ_2 = angle between refracted wave and optical axis, θ_3 = angle between reflected wave and optical axis, n_1 = refraction index of first medium, and n_2 = refraction index of second medium.

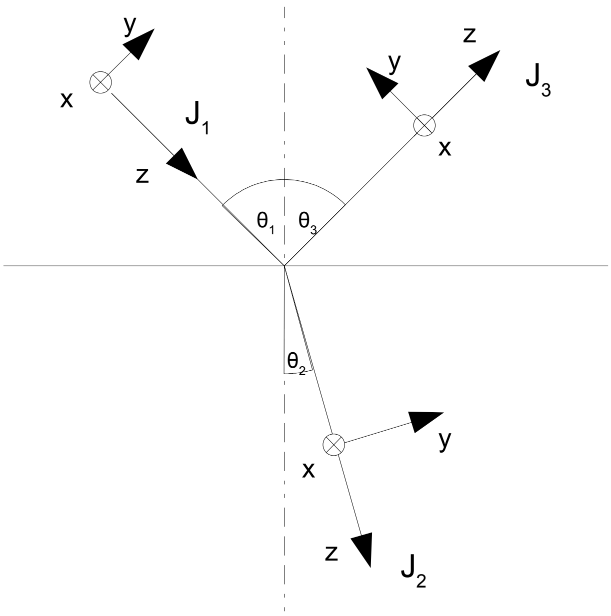

Fig. 2.A – Schematics of boundary effects

The boundary between materials can be considered as polarization device with different parameters for reflection (r) and transmission (t). For our purposes, we shall demonstrate these properties only for isotropic samples, so the Jones matrix will be diagonal. Waves polarized in x-axis are called transverse-electric or “senkrecht” (s), form German for perpendicular, waves in y-axis are called transverse-magnetic or parallel (p). The complex Fresnel coefficients t_s, t_p, r_s, r_p in Jones matrices

$$t=\begin{bmatrix} t_s & 0 \\ 0 & t_p \end{bmatrix};\quad r=\begin{bmatrix} r_s & 0 \\ 0 & r_p \end{bmatrix}, \tag{2.26}$$

can be determined from Snell's law and boundary conditions for Maxwell's equations

$$\begin{aligned} r_s &= \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}, \qquad t_s = 1 + r_s, \\ r_p &= \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}, \qquad t_p = \frac{n_1}{n_2}(1 + r_p). \end{aligned} \tag{2.27}$$

2.2.g) Degree of polarization, Polarization index

As mentioned in chapter 3.3c), we do not always work with completely polarized light, which is described by Jones vector. Some effects in optical components may cause the light to stop being completely polarized and rather depolarize the light, in other words, decrease its polarization factor[9]. Polarization factor (degree of polarization, *DOP*) is determined as the ratio of intensity of polarized light to total intensity. *P* has values from 0 (completely unpolarized light) to 1 (totally polarized light).

$$DOP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \tag{2.28}$$

Moreover, we want to know whether certain optical component depolarizes and if so, then how. The clear definition of how the component described by the Mueller matrix depolarizes varies in several publications. We will use the polarization index *PI* defined by [10,11]

$$PI = \frac{\sqrt{\mathrm{tr}(\boldsymbol{M}^{\boldsymbol{T}}\boldsymbol{M}) - 3\,m_{00}^2}}{3\mathrm{m}_{00}^2}, \tag{2.29}$$

where the symbol “tr” stands for matrix trace. The values of *PI* are the same as for *DOP,* 1 stands for non-depolarizing element, 0 stands for ideal depolarizer, i.e the diagonal matrix (1,0,0,0).

2.3. Depolarization

As non-depolarizing device (sample) is considered such component, that the degree of polarization of output light (DOP_o) is either greater or equal to the degree of polarization of incident light DOP_i, (i.e. $DOPo \geq DOPi$). Therefore, depolarizing device somehow lowers the degree of polarization. In our experiment, we shall postulate, that if depolarization occurs, it comes only from sample and not from apparatus. So we shall look into, what causes depolarization in reflection measurement from samples.

In general we can say that our incident light is divided and some part of it does not interfere with the rest and has different polarization. In other terms, the light looses its coherency. Depolarization can happen in many occasions:

Turbid layer with scattered particles can function as depolarizer, because the light can reflect many times from different grains and looses its coherency.

Thickness variation can be also a factor in poorly prepared samples. If the thickness is not uniform, the light reflects and transmits by variation of angles and lowers its DOP.

Reflection from different areas can be a problem with samples smaller than our beam of light and the detected intensity comes for example from the pedestal, upon which the measured sample is placed.

The use of quasi-monochromatic light can eventually lead to depolarization for samples that are non-depolarizing in monochromatic light. The wider halfwidth of the light causes shorter coherency length so all the other depolarizing effects appear sooner. the coherency length l_c is defined as

$$2k_{zj} = k_0 l_c, \tag{2.30}$$

where k_{zj} (j=1,2,3,4) are normal components of wavevectors, d is layer thickness, and $k_0 = 2\pi/\lambda$.

Thick substrate is one of the most frequent cases of depolarizing sample. The light transmitted by the upper boundary travels through the substrate a reflects from its lower boundary and travels back to the surface.

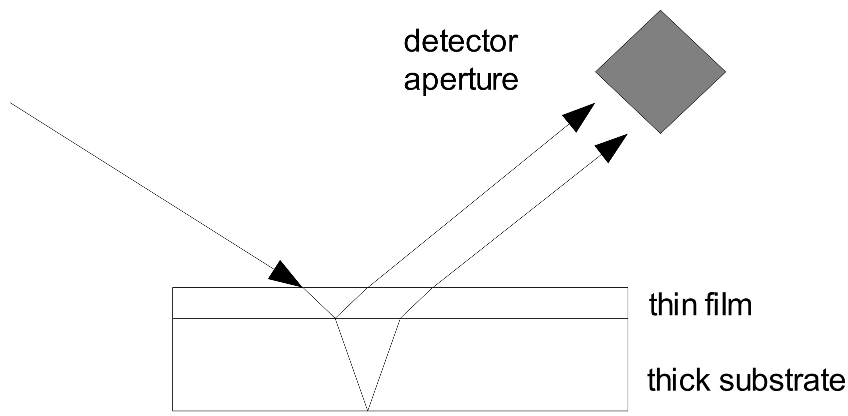

Fig. 2.B – reflection from back boundary

In the case of thin film, the light reflected from the bottom interferes with the light reflected from the top and the *DOP* does not change. However, for thick substrates, the path length of light is longer than its coherence length and its polarization state does no longer contribute to the total polarization state, only the intensities sum up. There are several possibilities on how to get rid of the problem of thick substrate:

- We can roughen the back boundary, so the reflected light is scattered and does not reach the detector aperture. This however damages the sample.
- By the use of immersion oil of the same refraction index we can add another substrate under the thick substrate, so the reflected light travels in the direction of aperture, but is so shifted it can't be detected.

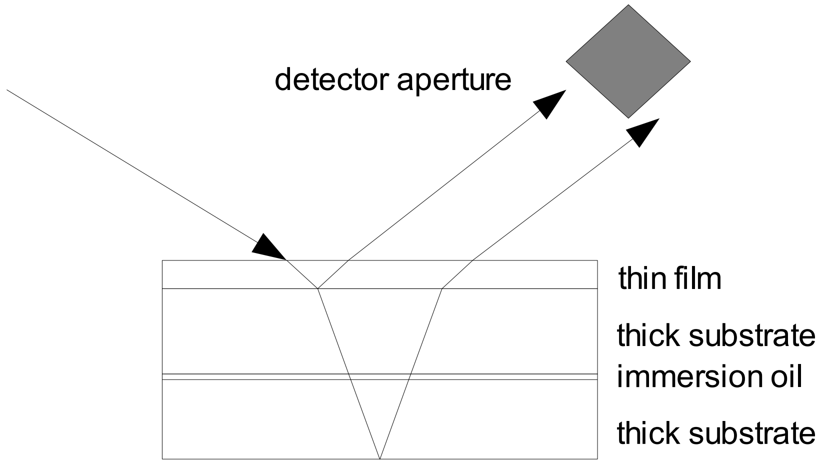

Fig. 2.C – reflection from very thick substrate

- Glue rough paper of substrate with immersion oil under the substrate – the light gets scattered. The use immersion oil does not physically damage the sample, however the sample needs cleaning afterward, which could the sample damage.

3. Spectroscopic ellipsometry

In this chapter we look into the measurement proposition and experimental setup of ellipsometer Uvisel. First, we describe the apparatus by Mueller matrix, define the measured quantities and suggest error minimalization by zone averaging.

3.1. Experimental setup

In our measurement, we use the ellipsometer Horiba Jobin – Yvon with spectral range from 0,6 eV – 6,5 eV / 190 nm - 2.1 μm / Near IR – Visible – Near UV light.

The optical components of this ellipsometer are aligned in this order: light source (Xe lamp) – polarizer – sample – modulator – monochromator – detector. The modulator consist of PEM and polarizer (so called analyzer) rotated by 45º.

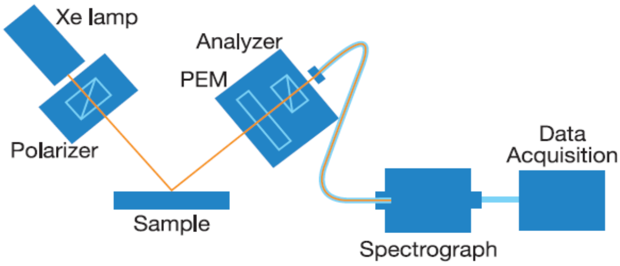

Fig 3.A – layout of the ellipsometer

There are two basic parameters, which directly influence the measurement, the angle of polarizer P and the angle of modulator M. The angles are determined as inclination from y-axis to x-axis. We do not change these parameters continuously, but rather use predefined values, i.e. zones. The values for parameter P are (0º, 45º, -º45) and for M are (0º, 45º, -45º, 90º). Alignment with P and M both equal 0º is used only for calibration of the apparature before measurement. The rest gives us the eight zones, in which we conduct our experiment.

3.2. Model of the sample

For our purpose, we shall use slightly different model of sample, than the ideal model. our model has these properties:

- Material forming the sample are isotropic and homogenous from the optical point of view.
- The boundaries are of the sharp (i.e the there are no transition layers).
- The ambient of the sample is homogenous, isotropic and non-absorbing from the optical point of view.

For non-depolarizing isotropic sample there are two ellipsometric angles Ψ and Δ defined. They are calculated from ratio between r_p and r_s

$$\begin{bmatrix} r_s & 0 \\ 0 & r_p \end{bmatrix} = r_{ss} \begin{bmatrix} 1 & 0 \\ 0 & \frac{r_p}{r_s} \end{bmatrix}, \tag{3.1}$$

$$\frac{r_p}{r_s} = \tan \Psi \, e^{i\Delta} = \chi , \tag{3.2}$$

Ψ denotes the azimuth and Δ denotes the phase change, respectively.

Jones matrix of isotropic sample is diagonal, the Mueller matrix is block-diagonal. The following elements are elements of Mueller-Jones matrix, i.e. non-depolarizing isotropic sample.

$$\begin{gathered} m_{00} = m_{11} = 1/2\left(|r_{ss}|^2 + |r_{pp}|^2\right) \\ m_{01} = m_{10} = 1/2\left(|r_{ss}|^2 - |r_{pp}|^2\right) \\ m_{22} = m_{33} = 1/2\left(r_{ss} r_{pp}{}^* + r_{pp} r_{ss}{}^*\right) \\ m_{23} = -m32 = i\, 1/2\left(r_{ss} r_{pp}{}^* + r_{pp} r_{ss}{}^*\right) \\ m_{02} = m_{03} = m_{12} = m_{13} = m_{20} = m_{21} = m_{02} = m_{31} = m_{32} = 0 \end{gathered} \tag{3.3}$$

3.3. Calculation of intensity

The description of optical system by matrix formalism allows us to calculate the detected intensity and shows us, how we can influence the measurement. Since we want to detect depolarization we must use the Mueller formalism.

Now we can calculate the outgoing intensity by multiplying the matrices. The incident light is described by Stokes vector:

$$\begin{bmatrix} \sin^2 P + \cos^2 P \\ \sin^2 P - \cos^2 P \\ 2 \sin P \cos P \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \pm 1 \\ 0 \end{bmatrix}_{P=\pm 45} \tag{3.4}$$

The sample by Mueller matrix:

$$\begin{bmatrix} m_{00} & m_{01} & 0 & 0 \\ m_{10} & m_{11} & 0 & 0 \\ 0 & 0 & m_{22} & m_{23} \\ 0 & 0 & m_{32} & m_{33} \end{bmatrix} \tag{3.5}$$

And the modulator by system of Mueller matrices describing the components and their rotation:

$$\underbrace{\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 2\cos^2(-M)-1 & 2\cos(-M)\sin(-M) & 0\\ 0 & -2\cos(-M)\sin(-M) & 2\cos^2(-M)-1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}_{[\boldsymbol{M}]^{-1}}\cdot\underbrace{\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}_{[\mathbf{45}]^{-1}}$$
$$\cdot\underbrace{\begin{bmatrix}1/2 & 1/2 & 0 & 0\\ 1/2 & 1/2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}}_{[\boldsymbol{A}]}\cdot\underbrace{\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}_{[\mathbf{45}]}\cdot\underbrace{\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & \cos\varphi & -\sin\varphi\\ 0 & 0 & \sin\varphi & \cos\varphi\end{bmatrix}}_{[\boldsymbol{PEM}]} \tag{3.6}$$
$$\cdot\underbrace{\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 2\cos^2 M-1 & 2\cos M\sin M & 0\\ 0 & -2\cos M\sin M & 2\cos^2 M-1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}_{[\boldsymbol{M}]}$$

Note: the first two matrices, $[\boldsymbol{M}]^{-1}$ and $[\mathbf{45}]^{-1}$, represent only the rotation of coordinate system back to our standard coordinate system and have no influence on intensity whatsoever, hence they are redundant in our calculation.

The detected intensity is the first parameter of the outgoing Stokes vector

$$\begin{aligned}I=&(1/2\,m_{00}+\cos\varphi\cos M\sin M\,m_{10})(\sin^2 P+\cos^2 P)\\ &+(1/2\,m_{01}+\cos\varphi\cos M\sin M\,m_{11})(\sin^2 P-\cos^2 P)\\ &+2(-1/2\cos\varphi(1-2\cos^2 M)m_{22}+1/2\sin\varphi\,m_{32})\sin P\cos P.\end{aligned} \tag{3.7}$$

The previous formula can be rewritten as

$$I=I_0+I_1\sin\varphi+I_2\cos\varphi, \tag{3.8}$$

where

$$I_0=1/2\,m_{00}(\sin^2 P+\cos^2 P)+(1/2\,m_{01})(\sin^2 P-\cos^2 P), \tag{3.9}$$

$$I_1=m_{32}\sin P\cos P, \tag{3.10}$$

and

$$\begin{aligned}I_2=&\cos M\sin M\,m_{10}(\sin^2 P+\cos^2 P)+\cos M\sin M\,m_{11}(\sin^2 P-\cos^2 P)\\ &+(1-2\cos^2 M)m_{22}\sin P\cos P.\end{aligned} \tag{3.11}$$

φ is oscillating function (Table 2.III), so we can write the trigonometric functions as first order

Bessel functions

$$\begin{aligned}\sin\varphi&=J_0(\varphi_A)\sin\varphi_0+2\mathrm{J}_1(\varphi_A)\cos\varphi_0\sin\omega t\\&+2\mathrm{J}_2(\varphi_A)\sin\varphi_0 cos2\omega t+\ldots,\\\cos\varphi&=J_0(\varphi_A)\cos\varphi_0-2\mathrm{J}_1(\varphi_A)\sin\varphi_0\sin\omega t\\&+2\mathrm{J}_2(\varphi_A)\cos\varphi_0 cos2\omega t+\ldots.\end{aligned}\tag{3.12}$$

Given that φ_0=0 we get

$$\begin{aligned}\sin\varphi&=2\mathrm{J}_1(\varphi_A)\sin\omega t+\ldots,\\\cos\varphi&=J_0(\phi_A)+2\mathrm{J}_2(\phi_A)\cos 2\omega t.\end{aligned}\tag{3.13}$$

We can see from Eq. (4.5) and (4.10) that I_0 is the unmodulated signal, I_1 is signal at ω, and I_2 is signal at 2ω (ω=50kHz).

P	M	I_0	I_1	I_2
0º	0º	$1/2\lvert r_{pp}\rvert^2$	0	0
45º	90º	$m_{00}/2$	$m_{32}/2$	$m_{22}/2$
45º	0º	$m_{00}/2$	$m_{32}/2$	$-m_{22}/2$
-45º	90º	$m_{00}/2$	$-m_{32}/2$	$-m_{22}/2$
-45º	0º	$m_{00}/2$	$-m_{32}/2$	$m_{22}/2$
45º	-45º	$m_{00}/2$	$m_{32}/2$	$m_{10}/2$
45º	45º	$m_{00}/2$	$m_{32}/2$	$-m_{10}/2$
-45º	-45º	$m_{00}/2$	$-m_{32}/2$	$m_{10}/2$
-45º	45º	$m_{00}/2$	$-m_{32}/2$	$-m_{10}/2$

Table 3.I – Detected intensities for different zones

There are few conclusions from the Table 3.I: The zone 0º, 0º gives us only unmodulated signal, the signals I_0 and I_1 are dependent only on angle of polarizer and the signal I_2 gives us two different values, depending on the angle of modulator. In phase modulated ellipsometry, the output quantities are ratios between the modulated and unmodulated signal

$$\begin{aligned}I_S&=\frac{I_1}{I_0}=\frac{m_{32}}{m_{00}}\qquad I_C=\frac{I_2}{I_0}\\I_{C1}&=\frac{m_{22}}{m_{00}}\qquad I_{C2}=\frac{m_{10}}{m_{00}}\end{aligned}\tag{3.14}$$

For isotropic non-depolarizing samples, these ratios allow us to directly calculate the ellipsometric angles Ψ and Δ that carry the information about sample and are used in the fitting process. From Eq. (3.1-3) and (3.14) we are given the following formulas:

$$I_S = \sin 2\Psi \sin \Delta \,, \tag{3.15}$$

$$I_{C1} = \sin 2\Psi \cos \Delta \,, \tag{4.16}$$

$$I_{C2} = \cos 2\Psi \,, \tag{4.17}$$

$$\frac{I_S}{I_{C1}} = \tan 2\Delta \,. \tag{4.18}$$

For depolarizing sample are the previous equations unusable, because they are linked to the Jones or Mueller-Jones matrices, not Mueller matrices.

3.4. Zone averaging

Errors occur in every measurement. The random errors affect measurement only in several points and are caused by noises and disturbances. The systematic errors occur from bad calibration and affect every point of the measurement and can be eliminated or minimalized by so called zone averaging. It is easy to see from Table 3.I, that the detected signals are same for several zones. This provides extremely powerful instrument for error minimalization[12].

For I_S signal, the detected values should be identical for every zone, with respect to the notation, so the zone averaging is addition and subtraction of detected signals and division by the count of measured zones

$$I_S = \frac{I_S^{45,90} + I_S^{45,0} + I_S^{45,-45} + I_S^{45,45} - I_S^{-45,90} - I_S^{-45,0} - I_S^{-45,-45} - I_S^{-45,45}}{8} \,. \tag{3.19}$$

The values of I_C are identical in four zones, depending on the angle of modulator. The averaged values are calculated as follows:

$$I_{C1} = \frac{I_C^{45,90} + I_C^{-45,0} - I_C^{-45,90} - I_C^{45,0}}{4} \,, \tag{3.20}$$

$$I_{C2} = \frac{I_C^{45,90} + I_C^{45,0} - I_C^{-45,-45} - I_C^{-45,45}}{4} \,. \tag{3.21}$$

3.5. Polarization index

This section describes how to calculate the polarization index of the sample. First, we divide the Muller matrix by its first element and use the symetry from eq. 3.3

$$m_{00}\begin{bmatrix} 1 & m_{10}/m_{00} & 0 & 0 \\ m_{10}/m_{00} & 1 & 0 & 0 \\ 0 & 0 & m_{22}/m_{00} & m_{32}/m_{00} \\ 0 & 0 & -m_{32}/m_{00} & m_{22}/m_{00} \end{bmatrix}. \qquad (3.22)$$

From Eq. 3.14 is is easy to see that the elements of the Mueller matrix are equal to the measured signals

$$m_{00}\begin{bmatrix} 1 & I_{C2} & 0 & 0 \\ I_{C2} & 1 & 0 & 0 \\ 0 & 0 & I_{C1} & I_S \\ 0 & 0 & -I_S & I_{C1} \end{bmatrix}. \qquad (3.24)$$

The polarization index can be calculated as

$$PI=\sqrt{\frac{1}{3}+\frac{2}{3}(I_{C2}^2+I_{C1}^2+I_S^2)} \qquad (3.25)$$

From Eq. (3.25) we can determine whether on not the measured sample is depolarizing. A non-depolarizing sample would have the value *PI* very close to 1 whereas the depolarizing would have it somewhat lower.

4. Experimental verification

In this section we describe two measurement of thick SiO_2 glass sample (thickness 510 μm), one with depolarization caused by reflection of back side of the substrate, one with depolarization removed through roughening of the back side. Understanding depolarization effects in this substrate would be very useful, since these thick glasses can be coated with other different materials and thanks to the changed absorption of infrared light could work as environment friendly isolation windows.

The measurement was conducted in all 8 zones, at the angle of incidence 60° with energy range from 0,6 to 6,5 eV with 0,2 eV step. Figure 4.I show the degree of polarization for both measurements.

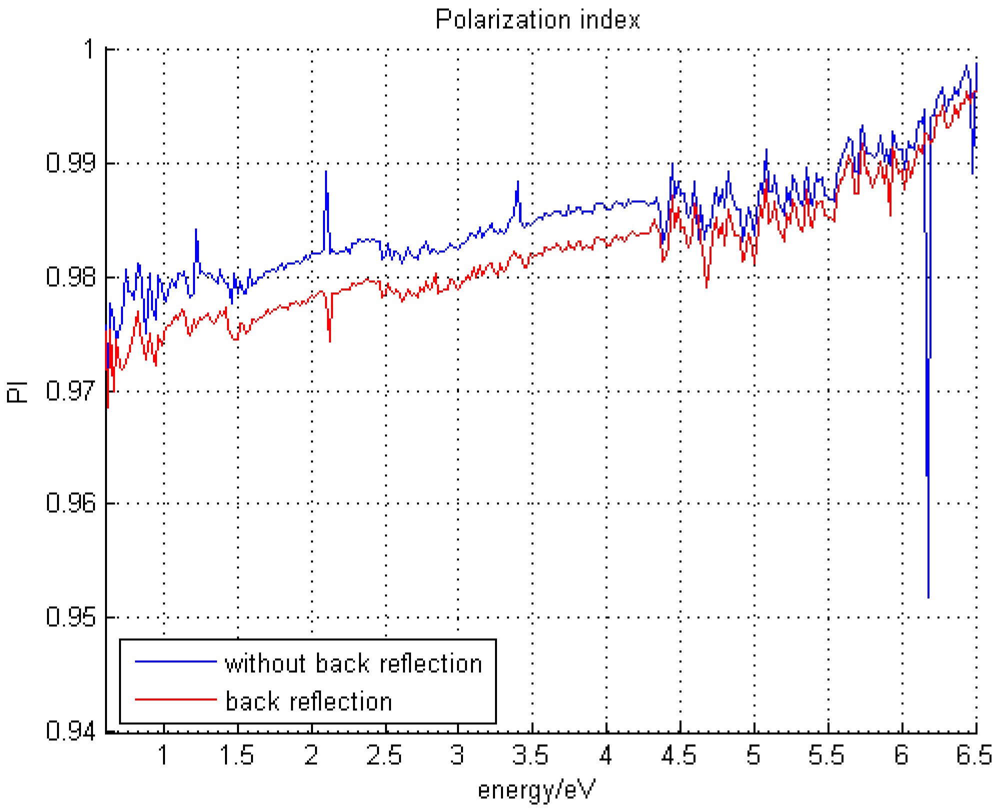

Fig. 4.1 – Polarization index for both measurements

From this figure we can see some trends in the development of the polarization index. First of all, the measurement without back reflection still shows depolarization of unknown origin. Second, both measurement shows relation to energy of photons, i.e. wavelength. This can be explained and modeled by dispersion of refractive index. And third, the difference of *PI* between measurement with and without back reflection is not so big, so the damaging of the sample by roughing the back surface is not necessary.

The next step is comparison the measured data with calculated model. The optical properties of the model are calculated from infinite incoherent summation of Fresnel coefficients for every boundary separated by thick layer (there are two such boundaries in our sample: air-SiO_2 and SiO_2-air). This procedure allows us to calculate reflection and transmission coefficients for the whole sample

$$<r_j r_k*>=[r_j^{(01)} r_k^{(01)}*]+\frac{[t_j^{(01)} t_k^{(01)}*][\tilde{t}_j^{(01)} \tilde{t}_k^{(01)}*][r_j^{(12)} r_k^{(12)}*]}{1-[\tilde{r}_j^{(01)} \tilde{r}_k^{(01)}*][r_j^{(12)} r_k^{(12)}*]}, \qquad (4.1)$$

$$<|r_j|^2> = |r_j^{(01)}|^2 + \frac{|t_j^{(01)}|^2 |\tilde{t}_j^{(01)}|^2 |r_j^{(12)}|^2}{1 - |\tilde{r}_j^{(01)}|^2 |r_j^{(12)}|^2}, \quad j, k = s, p. \tag{4.2}$$

where the tilde "~" denotes opposite direction, i.e. $\tilde{r}_s^{(01)} = r_s^{(10)}$.

Note: for absorbing sample we must add the absorbtion coefficient for losses in media, but SiO_2 is non-absorbing in used wavelengths.

These are the elements of diagonal coherency-transforming matrix

$$\begin{bmatrix} R_{11} & 0 & 0 & 0 \\ 0 & R_{22} & 0 & 0 \\ 0 & 0 & R_{33} & 0 \\ 0 & 0 & 0 & R_{44} \end{bmatrix} = \begin{bmatrix} <|r_s|^2> & 0 & 0 & 0 \\ 0 & <r_s r_p{}^*> & 0 & 0 \\ 0 & 0 & <r_p r_s{}^*> & 0 \\ 0 & 0 & 0 & <|r_p|^2> \end{bmatrix}, \tag{4.3}$$

from which can be the Mueller matrix calculated using Eq. (2.21) and its *PI* form Eq. (2.29).

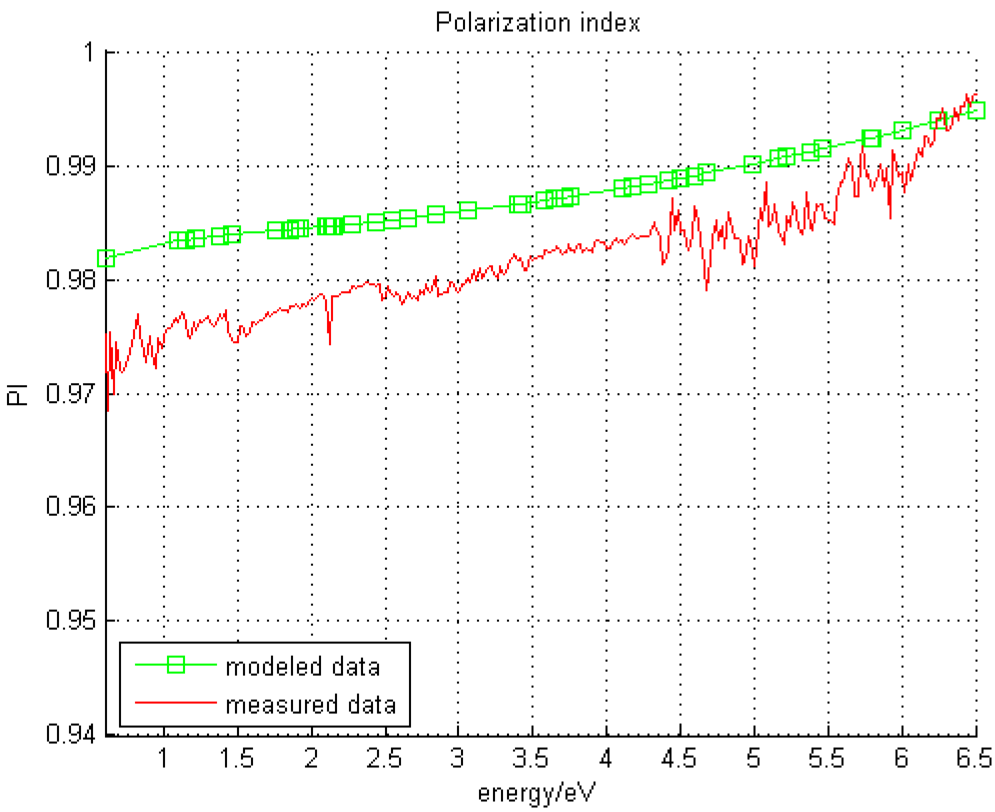

Fig. 4.II -modeled and calculated polarization index

Fig. 4.II shows the comparison of the modeled *PI* and *PI* calculated from measured data. As mentioned before, the trend of *PI* can be modeled depending on the index of refraction (each green square represents one known refraction index). The difference between modeled and

measured data is caused by the simplicity of the model, it does not include surface roughness, thickness variation or possible coating with thin layer.

The ellipsometric angles Ψ and Δ can be modeled in the same way as *PI*. The measurement gives us us three independent parameters, I_S, I_{C1}, I_{C2} from which we can calculate three characteristics *PI*, Ψ, and Δ.

5. Conclusion and perspectives

In this thesis were outlined the possibilities of detecting depolarization using phase modulated ellipsometry. For the mathematical description of polarized light were show the three matrix formalisms, the Jones matrices and vectors, the coherency-transforming matrices and the coherency vectors, and the Mueller formalism, and the Stokes vectors. For several light polarizations and optical components were shown their matrix representations. The degree of polarization and polarization index were defined as physical model necessary for description of depolarizing effects. The causes of depolarization and the possibilities of canceling depolarization effects were mentioned.

Spectroscopic ellipsometer was described by Mueller formalism and Stokes vector. It was shown that the detected intensity can be analyzed by second harmonic generation. The dependency of detected intensities on angles of modulator and polarizer was determined. From these formulas was shown how to calculate the the polarization index and the ellipsometric angles and how to minimize the error in the measured data by so called zone averaging.

The formulas postulated a determined in theoretical and mathematical sections were verified by measurement conducted on thick $Si0_2$ sample. The data were compared to the calculated model and it was show that it is possible for isotropic sample to predict the polarization index with simple model based on incoherent reflection from the back boundary.

This thesis is just a brief insight to the rather complex world of depolarization and outlines what more can be done in this area, for example:

- more complex modeling of depolarizing samples – by decomposition of Mueller matrices and by effective medium layer
- experimental verification of other methods that cancel the depolarization effect, other than roughening the back surface

- conduct measurement on more complex samples – isotropic, absorbing and comparison with models of such samples.

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